2.4 Exercises

In Exercises 1–6, complete the table.

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1. $y = (5x - 8)^4$		
2. $y = \dfrac{1}{\sqrt{x+1}}$		
3. $y = \sqrt{x^3 - 7}$		
4. $y = 3\tan(\pi x^2)$		
5. $y = \csc^3 x$		
6. $y = \sin \dfrac{5x}{2}$		

In Exercises 7–36, find the derivative of the function.

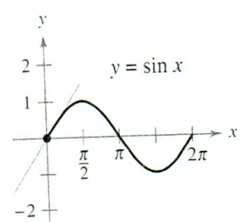
7. $y = (4x - 1)^3$
8. $y = 2(6 - x^2)^5$
9. $g(x) = 3(4 - 9x)^4$
10. $f(t) = (9t + 2)^{2/3}$
11. $f(t) = \sqrt{5 - t}$
12. $g(x) = \sqrt{9 - 4x}$
13. $y = \sqrt[3]{6x^2 + 1}$
14. $g(x) = \sqrt{x^2 - 2x + 1}$
15. $y = 2\sqrt[4]{9 - x^2}$
16. $f(x) = -3\sqrt[4]{2 - 9x}$
17. $y = \dfrac{1}{x - 2}$
18. $s(t) = \dfrac{1}{t^2 + 3t - 1}$
19. $f(t) = \left(\dfrac{1}{t - 3}\right)^2$
20. $y = -\dfrac{5}{(t + 3)^3}$
21. $y = \dfrac{1}{\sqrt{x + 2}}$
22. $g(t) = \sqrt{\dfrac{1}{t^2 - 2}}$
23. $f(x) = x^2(x - 2)^4$
24. $f(x) = x(3x - 9)^3$
25. $y = x\sqrt{1 - x^2}$
26. $y = \frac{1}{2}x^2\sqrt{16 - x^2}$
27. $y = \dfrac{x}{\sqrt{x^2 + 1}}$
28. $y = \dfrac{x}{\sqrt{x^4 + 4}}$
29. $g(x) = \left(\dfrac{x + 5}{x^2 + 2}\right)^2$
30. $h(t) = \left(\dfrac{t^2}{t^3 + 2}\right)^2$
31. $f(v) = \left(\dfrac{1 - 2v}{1 + v}\right)^3$
32. $g(x) = \left(\dfrac{3x^2 - 2}{2x + 3}\right)^3$
33. $f(x) = ((x^2 + 3)^5 + x)^2$
34. $g(x) = (2 + (x^2 + 1)^4)^3$
35. $f(x) = \sqrt{2 + \sqrt{2 + \sqrt{x}}}$
36. $g(t) = \sqrt{\sqrt{t + 1} + 1}$

CAS In Exercises 37–42, use a computer algebra system to find the derivative of the function. Then use the utility to graph the function and its derivative on the same set of coordinate axes. Describe the behavior of the function that corresponds to any zeros of the graph of the derivative.

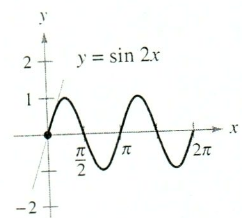
37. $y = \dfrac{\sqrt{x} + 1}{x^2 + 1}$
38. $y = \sqrt{\dfrac{2x}{x + 1}}$
39. $y = \sqrt{\dfrac{x + 1}{x}}$
40. $g(x) = \sqrt{x - 1} + \sqrt{x + 1}$
41. $y = \dfrac{\cos \pi x + 1}{x}$
42. $y = x^2 \tan \dfrac{1}{x}$

In Exercises 43 and 44, find the slope of the tangent line to the sine function at the origin. Compare this value with the number of complete cycles in the interval $[0, 2\pi]$. What can you conclude about the slope of the sine function $\sin ax$ at the origin?

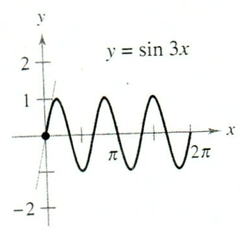
43. (a)



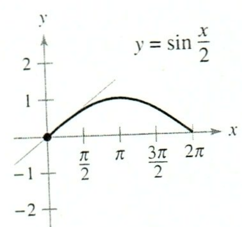
(b)



44. (a)



(b)



In Exercises 45–66, find the derivative of the function.

45. $y = \cos 4x$
46. $y = \sin \pi x$
47. $g(x) = 5\tan 3x$
48. $h(x) = \sec x^2$
49. $y = \sin(\pi x)^2$
50. $y = \cos(1 - 2x)^2$
51. $h(x) = \sin 2x \cos 2x$
52. $g(\theta) = \sec(\frac{1}{2}\theta)\tan(\frac{1}{2}\theta)$
53. $f(x) = \dfrac{\cot x}{\sin x}$
54. $g(v) = \dfrac{\cos v}{\csc v}$
55. $y = 4\sec^2 x$
56. $g(t) = 5\cos^2 \pi t$
57. $f(\theta) = \tan^2 5\theta$
58. $g(\theta) = \cos^2 8\theta$
59. $f(\theta) = \frac{1}{4}\sin^2 2\theta$
60. $h(t) = 2\cot^2(\pi t + 2)$
61. $f(t) = 3\sec^2(\pi t - 1)$
62. $y = 3x - 5\cos(\pi x)^2$
63. $y = \sqrt{x} + \frac{1}{4}\sin(2x)^2$
64. $y = \sin\sqrt[3]{x} + \sqrt[3]{\sin x}$
65. $y = \sin(\tan 2x)$
66. $y = \cos\sqrt{\sin(\tan \pi x)}$

In Exercises 67–74, evaluate the derivative of the function at the given point. Use a graphing utility to verify your result.

67. $s(t) = \sqrt{t^2 + 6t - 2}$, $(3, 5)$
68. $y = \sqrt[5]{3x^3 + 4x}$, $(2, 2)$
69. $f(x) = \dfrac{5}{x^3 - 2}$, $\left(-2, -\dfrac{1}{2}\right)$
70. $f(x) = \dfrac{1}{(x^2 - 3x)^2}$, $\left(4, \dfrac{1}{16}\right)$
71. $f(t) = \dfrac{3t + 2}{t - 1}$, $(0, -2)$
72. $f(x) = \dfrac{x + 1}{2x - 3}$, $(2, 3)$
73. $y = 26 - \sec^3 4x$, $(0, 25)$
74. $y = \dfrac{1}{x} + \sqrt{\cos x}$, $\left(\dfrac{\pi}{2}, \dfrac{2}{\pi}\right)$